

## Linear Harmonic Motion

### Object:

To study the oscillatory motion of a mass suspended by a spiral spring.

### Apparatus:

Mirror scale mounted on a spring-support stand; light spring with weight holder; slotted weights in increments of 10, 20, 50 and 100 gm, Science Workshop™ Interface, motion sensor, motion sensor protector, large spring-support stand.

### Theory:

A mass  $M$  will execute simple harmonic motion, if its displacement “ $x$ ” from an equilibrium position, is opposed by a restoring force  $F$  that is proportional to the displacement but in the opposite direction i.e. if:

$$\begin{aligned}\vec{F} &\propto -\vec{x} \\ \text{or} \\ \vec{F} &= -k\vec{x}\end{aligned}\tag{1}$$

where  $k$  is a constant of proportionality. The direct proportion between  $\vec{F}$  and  $\vec{x}$  is known as Hooke’s law and  $k$  is called the spring constant. As used here,  $k$  will be in *Newton/m*. If  $f$  is the frequency of vibration (in vibrations per sec), the reciprocal of  $f$  is the number of seconds per vibration, which is called the period “ $T$ ” of the motion. Obviously, then, we can write

$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}\tag{2}$$

If the motion were circular, we could associate an angular velocity  $\omega$  with the frequency, as was done in the experiment on uniform circular motion. In that case, we would have:

$$\omega = 2\pi f \quad \text{or} \quad \omega = \frac{2\pi}{T}\tag{3}$$

From this, we can get:

$$T = \frac{2\pi}{\omega}\tag{4}$$

A somewhat more sophisticated study of this motion reveals that:

$$\omega = \sqrt{\frac{k}{M}}\tag{5}$$

from which we can write:

$$T = 2\pi\sqrt{\frac{M}{k}}\tag{6}$$

**Procedure:**

**Part A.** Obtain data to verify Hooke's law. In the table, record the scale reading of displacement of the spring as the 10, 20 50 and 100 gm masses are added to the weight holder. Plot load vertically vs. displacement horizontally.

Total Mass (kg)	Force (N)	Displacement (m)

**Analysis:** Interpret the Displacement vs. Force graph. Determine the slope and state its units. Deduce the spring constant from the slope of your graph.

**Part B.** In this part of the activity, a motion sensor measures the motion of a mass suspended from the end of the spring. The *Science Workshop* program records the motion and displays position and velocity of the oscillating mass. The period of oscillation is measured and compared to the theoretical value.

**I : Computer Setup**

1. Connect the *Science Workshop* interface to the computer, turn on the interface, and turn on the computer.
2. Connect the motion sensor's stereo phone plugs into Digital Channels 1 and 2 of the interface. Plug the yellow-banded (pulse) plug into Digital Channel 1 and the second plug (echo) into Digital Channel 2.
3. Open the Science Workshop document titled as shown:  
Windows: P19\_MASS  
\* The Science Workshop document will open with a Graph display with plots for Position (m) and Velocity (m/s) versus Time (sec).

**II: Sensor Calibration and Equipment Setup**

- \* You do not need to calibrate the motion sensor. Make sure the slide switch on the top of the motion sensor is in the "cart" position not the "person" position.
1. Using a support rod and clamp, suspend the spring so that it can move freely up-and-down. Put a mass hanger on the end of the spring.
  2. Add 50-100 gm of mass to the hanger.
  3. Record the total mass (in kilograms) in the Data section.
  4. Place the motion sensor on the desk directly beneath the mass hanger, and cover it with a protector. (Make sure that the motion sensor is not tilted inside the protector.)
  5. Adjust the position of the spring so that the minimum distance from the mass hanger to the motion sensor is approximately 30 cm at the bottom of the mass hanger's movement.

### III: Data Recording

1. Pull the mass down to stretch the spring about 2 cm. Release the mass. Let it oscillate a few times so the mass hanger will move up-and-down without much side-to-side motion.
3. Click the “start” button to begin recording data.
4. The plots of the position and velocity of the oscillating mass will appear in the Graph display.

**Troubleshooting Note:** If the data points do not appear on the plots for position and velocity, click on the “Autoscale” button to rescales the graph. The position curve should resemble the plot of a sine function. If it does not, check the alignment of the Motion Sensor and the bottom of the mass hanger at the end of the spring. You should try several different position of motion sensor to get better graphs. Increasing or decreasing mass may help you to get better graphs. To erase a run of data, select the run in the Data list and press the “Delete” key.

### Analysis:

1. Click the “Autoscale” button to rescale the Graph display.
2. Click the “Smart Cursor” button. The cursor changes to a cross-hair when you move it into the display area of the graph. The X- and Y-coordinates of the cursor’s position are shown next to the horizontal and vertical axes.
3. To find the average period, calculate the difference between the time for several waves, then divide it by the number of waves.

Number of waves  $n = \underline{\hspace{2cm}}$

Time difference  $\Delta t = \underline{\hspace{2cm}}$  (sec)

Measured period of oscillation  $= \frac{\Delta t}{n} = \underline{\hspace{2cm}}$  (sec)

### Calculation:

Calculate the theoretical value for the period of oscillation based on the measured value of the spring constant of the spring and the mass on the end of the spring. (Use mass in kilograms for this calculation).

Mass =                      kg

$$T = 2\pi \sqrt{\frac{M}{k}}$$

Calculated period of oscillation =                      sec

### Questions:

1. How does your calculated value for the period of oscillation compare to the measured value for the period of oscillation? Find the percent difference between your calculated value and the measured value.

$$\text{Mean} = \frac{\text{calculated} + \text{measured}}{2}$$

$$\text{Percent difference} = \left| \frac{\text{calculated} - \text{measured}}{\text{mean}} \right| \times 100\%$$

2. When the position of the mass is farthest from the equilibrium position, what is the velocity of the mass? (Hint: Choose two or three oscillations by Magnifier button. Then, move the Smart Cursor to a peak on the position plot, hold down the Shift key, and move the Smart Cursor onto the velocity plot. The velocity will be displayed next to the vertical axis.)
3. When the absolute value of the velocity of the mass is greatest, where is the mass relative to the equilibrium position?