2003

COMPREHENSIVE EXAMINATION
Department of Physics
University of Nevada, Reno

January 13, 2003
9:00-11:30 AM

MATHEMATICAL PHYSICS

Answer any four problems. Do not turn in (partial) solutions for more than four problems. Each problem has the same weight.

(1) (*a*) Show that the function

$$u(x,y) = e^{-x}(x \sin y - y \cos y).$$

is harmonic.

(*b*) A curve C is given by $y = x^3 - 3x^2 + 4x - 1$ connecting the points (1,1) and (2,3). Find the value of the definite path integral

$$\int_C (12z^2 - 4iz)dz.$$

(2) Using contour integration, evaluate the integral

$$\int_0^\infty \frac{\sin x}{x} dx.$$

(3) Find the Laurent series of the following function

$$f(z) = \frac{z}{(z+1)(z+2)}$$

about $z = -2$ in all regions of convergence.

(4) (a) Show that a necessary and sufficient condition that $u(x,y,z)$ and $v(x,y,z)$ are connected by some function $f(u,v) = 0$ is that $(\nabla u) \times (\nabla v) = 0$.

(b) If $u = u(x,y)$ and $v = v(x,y)$, show that the condition $(\nabla u) \times (\nabla v) = 0$ leads to the two-dimensional Jacobian

$$J\left(\frac{u,v}{x,y}\right) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = 0.$$

The functions u and v are assumed differentiable.

(5) If $S(x,y,z) = (x^2+y^2+z^2)^{-3/2}$, find (a) $\vec{\nabla} S$ at the point (1,2,3); (b) the magnitude of the gradient of S, $\left|\vec{\nabla} S\right|$ at (1,2,3); and (c) the direction cosines of $\vec{\nabla} S$ at (1,2,3).

COMPREHENSIVE EXAMINATION
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January 13, 2003
2:30 – 5:00 PM

CLASSICAL MECHANICS

Answer any four problems. Do not turn in solutions for more than four problems. All problems have the same weight.

1. Starting from Newton's second law and considering a central force F, show that the equations of motion for a particle of mass m can be written in two-dimensional polar coordinates r and θ in the following form,

$$m\left(\frac{d^2r}{dt^2}-r\left(\frac{d\theta}{dt}\right)^2\right)=f(r) \quad \text{and} \quad m\left(2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)+r\frac{d^2\theta}{dt^2}\right)=0,$$

where $f(r)$ is the magnitude of the central force F,

$$F=f(r)\, e_r .$$

Integrate these equations explicitly and show that the total mechanical energy and angular momentum are conserved.

2. A mass point m moves along the x-axis. The distance x is related to the velocity according to,

$$x=\frac{1}{2a}\log\left(\frac{b+av_o^2}{b+av^2}\right)$$

where a and b are constants, and v_o is the velocity for $x=0$. (a) Determine the force F under which the mass point moves. (b) Is this force conservative?

3. Show that (a) if the Lagrangian L does not depend explicitly on time then the Hamiltonian H is a constant of the motion, and (b) if there are only time-independent constraints and potentials (i.e. $V=V(x_1,x_2,\dots,x_N)$) then the Hamiltonian is constant and equal to the total energy E of the system. Justify all your steps.

4. A rocket of mass m is in a circular orbit with radius R about a planet of radius R_o. (a) What tangential impulse will cause the rocket to gaze the backside of the planet? (b) Describe the resulting orbit.

5. A string of uniform mass density σ and length l hangs under its own weight in the earth's gravitational field. Consider small transverse displacements from equilibrium $u(x,t)$ in a plane.

(a) Compute the equilibrium tension $\tau(x)$, where x is the distance from the point of suspension.

(b) Estimate the frequency and eigenfunction of the lowest normal mode of oscillation of the string using a variational method and the Rayleigh-Ritz approximation.

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ELECTRICITY AND MAGNETISM

Answer any four problems. Do not turn in solutions for more than four problems. All problems have the same weight.

1. Consider a hollow cylinder of radius a and length L with uniform surface charge σ, and rotating about its axis (i.e. the z-axis) with constant angular velocity ω.

   (a) Find out the distribution of current density J, and the dipole moment of J.
   (b) Calculate the magnetic field B for any point on the z-axis.
   (c) Take the limit of B for large z and interpret the result.
   (d) Consider another cylinder of the same size but with a uniform magnetization M_o parallel to the z-axis. Determine the distribution of magnetization current density J_M and magnetic "charge" density ρ_M. Draw a scheme of the B and H magnetic field lines.

2. Prove the *mean value theorem*: for charge-free space the value of the electrostatic potential at any point is equal to the average of the potential over the surface of *any* sphere centered on that point.

3. Two point-like charges q and $-q$ are located on the z-axis at $z=+a$ and $z=-a$, respectively.

   (a) Find the electrostatic potential as an expansion in spherical harmonics and powers of r for $r > a$ and $r < a$.
   (b) Keeping the product $qa=p/2$ constant, take the limit of $a \to 0$ and find the potential for $r \neq 0$.
   (c) For (b), consider the additional effect of a grounded spherical shell of radius b concentric with the origin. Find out the potential inside the shell.

4. A uniformly charged spherical shell of internal and external radii a and b, and charge per unit of volume ρ rotates with constant angular velocity ω about an axis that goes through the geometrical center of the shell.

(a) Determine the magnitude and direction of the volume current density J associated with the rotating shell.
(b) Calculate the magnitude and direction of the magnetic dipole moment m of the volume current density J.
(c) Find out the magnitude and direction of the vector potential A of the magnetic field B produced by the rotating shell, for all space points.

5. Use Maxwell equations to work out the theorem of conservation of angular momentum for the electromagnetic field. Justify all your steps.

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THERMAL AND STATISTICAL PHYSICS

Answer any three problems. Do not turn in (partial) solutions for more than three problems. Each problem has the same weight.

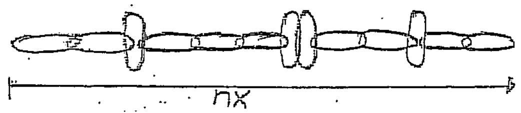
(1) Given a molecule with the three lowest energy levels at $E_1 = 0$, $E_2 = \varepsilon$, $E_3 = 10\varepsilon$.

(a) Find the temperature T_c below which only the levels E_1 and E_2 are populated in a system consisting of a large number N of such molecules.(Hint: Start with the ratios N_2/N_1 and N_3/N_1 where the N_i refer to the number of molecules at energies E_i, respectively .)

(b) Find the average energy E of the molecule at temperature T.

(c) Find the molar specific heat C_v and discuss low and high temperature limits. The use of Maxwell - Boltzmann statistics is appropriate.

(2) A model of a one-dimesional chain with n links ($n \gg 1$) is given as depicted



i.e. the length of each horizontal segment is a and of each vertical segment *zero* . The distance between the ends of the chain is nx .

(a) Find the entropy of the chain as a function of x.

(b) Assuming the joints turn freely from the vertical position at energy *zero* to the horizontal position at energy Fa (there are only horizontal and vertical positions possible!) , obtain a relation between the temperature T and the tension F which is necessary to maintain the distance nx.

(c) Does the answer in (b) satisfy Hooke's law ?

(3) Suppose that two systems are identical in every respect except that the energy levels of one have each been shifted by an arbitrary amount Δ relative to the other. As a result, the partition functions for the two systems can be generally written,

$$Q = \sum_j e^{-\beta E_j}$$

$$Q' = \sum_j e^{-\beta(E_j+\Delta)}$$

(a) Show that both the average energy $\langle E \rangle$ and the Helmholtz free energy of the primed system are shifted by Δ relative to the unprimed system.

(b) Show that the entropy, S, is the same for both systems.

(c) Show that the chemical potential μ and the pressure P do not depend on the value of Δ.

4. A composite isolated system consists of 1mole of monatomic ideal gas ($E_1 = \frac{3}{2}N_1RT$) and 2 moles of a diatomic ideal gas ($E_2 = \frac{5}{2}N_2RT$), separated into subsystems of equal volume by a fixed, impermeable,diathermal wall.

(a) Given the total energy E and the total volume V, derive expressions for the equilibrium temperature and pressure of each subsystem in terms of known quantities.

(b) The wall separating the two subsystems is released, allowing the system to reach a new equilibrium. Calculate the consequent change in total entropy.

COMPREHENSIVE EXAMINATION
DEPARTMENT OF PHYSICS
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January 17, 2003
2:30 - 5:00 PM

Modern Physics

Try to give a definition, a short explanation or an example of all of the physics terms listed below. Then expand on some (or all) of them, so that the result might serve e.g. as an entry in a dictionary of physics.

a) Coherent state of a quantum oscillator.
b) Irreversible processes.
c) Nuclear fission.
d) String theory
e) Zero point energy.
f) Terminal velocity of a falling body.
g) Quantum dots and nano tubes.
h) Total reflection.
i) Enthalpy.
j) Breeding reactor.
k) Diffusion equation.
l) Extinction coefficient.
m) Osmosis and osmotic pressure.
n) Surface tension.
o) Dispersion relations
p) Anti - atoms.
q) plasma frequency
r) solar neutrino problem

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QUANTUM THEORY

Answer any four problems. Do not turn in solutions for more than four problems.

(1) Two identical linear oscillators with spring constant k interact via a potential $H' = cx_1x_2$ where x_1 and x_2 are the oscillator variables.

(a) Find the exact energy levels.

(b) Assume $c \ll k$ and calculate the lowest excited energy level in first order of perturbation theory. Compare your result to (a).

(2). The following Hamiltonian matrix is given

$$H = \begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} & \dots \\ H_{21} & H_{22} & 0 & 0 & \dots \\ H_{31} & 0 & H_{33} & 0 & \dots \\ H_{41} & 0 & 0 & H_{44} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

This matrix in arrow form can be diagonalized in closed form to obtain an exact value for the eigenenergy E_1. Compare the result of the diagonalization to the energy formula in second order.

(3) Consider a physical system whose 3D state space is spanned by the orthonormal basis formed by the three kets $|u_1>$, $|u_2>$, and $|u_3>$. In the basis of these three vectors (taken in this order) two operators H and B are defined by:

$$H = \hbar\omega \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad B = b \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

where ω and b are real constants.

(a) Are H and B Hermitian ?

(b) Show that H and B commute. Give a basis of eigen vectors common to both of the operators.

(c) Of the sets of operators: $\{H\}, \{B\}, \{H, B\}, \{H^2, B\}$, which form a complete set of commuting observables (CSCO) ?

(4) Determine the expectation values $\langle X \rangle (t)$ and $\langle P \rangle (t)$ of a charged particle moving in a homogeneous electric field. Here X and P indicate the position and momentum operators, respectively.

(5) Prove the following identity:

$$\sum_n \frac{2m}{\hbar^2} |\langle n|\hat{x}|0\rangle|^2 (E_n - E_0) = 1.$$

The sum is over the complete set of eigenkets $|n\rangle$ of energy E_n of a particle of mass m, which moves in a potential. The ket $|0\rangle$ designates a *bound* state, not necessarily the ground state. The problem as stated describes a one-dimensional system.