

COMPREHENSIVE EXAMINATION

Department of Physics

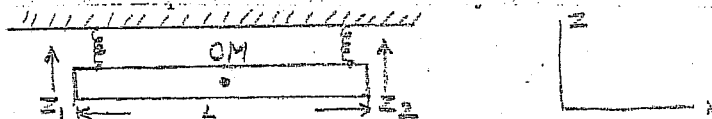
University of Nevada, Reno

January 10, 2005; 2:30-5:00 PM

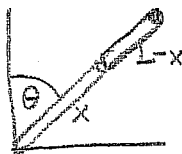
CLASSICAL MECHANICS

Answer any four problems. Do not turn in solutions for more than four problems.

1. A rigid uniform bar of mass M and length L is supported in equilibrium in a horizontal position by two identical, massless springs of force constant k . The motion of the center of gravity is constrained to move parallel to the vertical z -axis. Furthermore the motion of the object is constrained to the xz -plane. Find the normal modes and frequencies of small vibration of the system.



2. A thin uniform chimney is pivoted at its low end. Show that a section through the chimney at any point undergoes a flexion stress, and calculate the most probable point of rupture as the chimney falls.



3. The free surface of a liquid is one of constant pressure. An incompressible fluid is placed in a cylindrical vessel and the whole rotated with constant angular velocity ω . Show that the free surface becomes a paraboloid of revolution.

THERMAL AND STATISTICAL PHYSICS

Answer all four problems. Each problem has the same weight.

- 1.(a) Derive the heat conduction equation

$$\frac{\partial T}{\partial t} = \chi \nabla^2 T, \quad \chi = \text{const.}$$

(b) A thin hot wire, kept at a constant temperature T_0 is immersed in a heat conducting medium. How does the temperature decrease as a function of r , the distance from the wire?

2. Show that for a Carnot cycle engine with energy loss to its environment

$$\oint \frac{dQ}{T} < 0.$$

3. Compute the increase in entropy by the mixing of two identical fluids with temperature T_1, T_2 and masses M_1, M_2 .

4. Explain the differences between Boltzmann, Bose and Fermi statistics.

COMPREHENSIVE EXAMINATION

Department of Physics
University of Nevada, Reno

January 14, 2005
9:00-11:30 AM

QUANTUM THEORY

Answer any four problems. Do not turn in solutions for more than four problems.

1. The following Schrödinger eigenvalue equation describes a charged harmonic oscillator in a uniform electric field

$$\left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{m}{2} \omega^2 x^2 - qEx \right) \Phi(x) = W\Phi(x)$$

Determine the eigenenergies and eigenvectors.

This problem can be used as a simple model for induced dipole moments of atomic electrons which are elastically bound to their ions. Discuss the consequences of an external electric field on the susceptibility of the system.

2. Determine the coefficient of reflection for a one-dimensional potential step:

$$\begin{aligned} V &= 0 \text{ for } x < 0 \text{ and} \\ V &= V_0 \text{ for } x \geq 0. \end{aligned}$$

The energy of the particle impinging from the left (i.e. the direction of the negative x -axis) is greater than V_0 .

3. Given is a system of two *non-identical* spin $\frac{1}{2}$ particles. \vec{S}_1 and \vec{S}_2 are the corresponding observables. A measurement of the y - component of \vec{S}_2 (i.e. $S_{2,y}$) is to be performed.

- (a) What is the probability of obtaining $\hbar/2$?
(b) In a second measurement the spin of both particles is determined. What is the probability that the total system is spinless?

4. Calculate energies and wave functions of the hydrogen atom using the variational method and assuming the following variational functions:

(a) $\psi_{1s} = Ae^{-\beta r}$ for the ground state.

(b) $\psi_{2s} = B \left(1 + \frac{\gamma r}{a_0} \right) e^{-\alpha r/a_0}$ for the $2s$ state.

- (c) What would be a reasonable form of ψ for the $2p$ state and how does the variational procedure differ from the one to be used in part (b)?

HINTS: A and B are normalization constants, not variation parameters.

The constant $a_0 = \hbar^2/2\mu$ is not a variational parameter either, but α and γ are.

The following integral formula may be helpful $\int_0^\infty x^n e^{-Cx} dx = \frac{n!}{C^{n+1}}$ for $C > 0$ and integer values of n .

Using atomic units, the hydrogen atom Hamiltonian reads as $\hat{H} = -\frac{\hbar^2}{2\mu} \nabla^2 - \frac{e^2}{r}$.

5. Determine the expectation values $\langle X \rangle(t)$ and $\langle P \rangle(t)$ of a charged particle moving in a homogeneous electric field. Here X and P indicate the position and momentum operators, respectively and $\langle \dots \rangle(t)$ indicates the time-dependent expectation values.

COMPREHENSIVE EXAMINATION

Department of Physics

University of Nevada, Reno

January 14, 2005; 2:30-5:00 PM

Modern Physics

Provide answers to the following questions using complete sentences, aided by sketches or equations as necessary. All questions carry equal weight. Supply any information that you consider relevant to the topic.

1. Outline the physical principles upon which a laser operates, emphasizing the conditions that must be satisfied for laser action. Use of a specific or generic example may be helpful.
2. Outline the basic premises upon which the special theory of relativity is based, as well as the main consequences.
3. Outline the basic premises upon which the general theory of relativity is based, as well as the main consequences.
4. Characterize the physical similarities and differences between electrical insulators, semiconductors and conductors in terms of their electronic structures. Give an example of a semiconductor device and describe its operating principle.
5. Outline the basis for a Nobel Prize in physics that was awarded during the last decade (1994-2004).

COMPREHENSIVE EXAMINATION

Department of Physics

University of Nevada, Reno

January 12, 2005; 9:00-11:30 AM

ELECTRICITY AND MAGNETISM

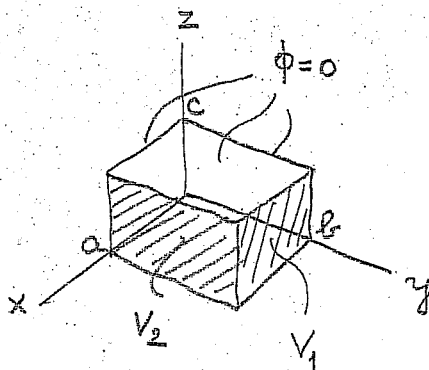
Answer any three problems. Do not turn in solutions for more than three problems.

1. (a) A conducting box of dimensions a, b and c as shown below has all faces at zero potential, except for the right plate which is held at potential $\Phi_1 = V_1(x, z)$. Determine the potential everywhere inside the box. Assume the following functional form

$$V_1 = \sin\left(\frac{2\pi}{a}x\right)\sin\left(\frac{3\pi}{c}z\right).$$

(b) Now assume that *in addition* the front plate is held at potential $\Phi_2 = V_2(y, z)$ and design a strategy to solve the augmented problem.

(c) Assume finally that $V_2 = \cos(By)\cos(Cz)$. How do you have to choose B and C so that V_2 is compatible with the rest of the boundary conditions?



2. A point charge q sits on the interface between two dielectric media. Find the fields \vec{E} , \vec{D} , \vec{P} and surface charge density (if any).

3. (a) Using the method of images, find the potential of a dipole placed at an angle near a large, grounded, conducting surface (only the $z > 0$ region is of interest here).

(b) Determine the magnitude and direction of the dipole moments of both the original dipole and the image.

(c) The force exerted by the dipole on the plane is equal to the force exerted by the dipole on its image. Calculate the force for the case that the dipole is perpendicular to the surface.

4. Assume that an electron introduced into a uniform magnetic field $\hat{z}B_0$ with constant velocity $\hat{x}v$ moves in a circle of radius r . What is the current? The velocity in the field? The angular velocity? The work done on the electron by the magnetic field? The change in kinetic energy?

5. A very long cylinder of radius R is magnetized with the magnetization given by

$$\vec{M} = (p\rho \sin 2\phi + q\rho \cos \phi)\hat{\rho} + \left(\frac{1}{2}p\rho \cos 2\phi - q\rho \sin \phi\right)\hat{\phi}.$$

Here p and q are constants. Find the inside and outside potentials. (Hint: This problem can be treated using the magnetic *scalar* potential, i.e. Φ_m satisfies Laplace's equation. Why?)

MATHEMATICAL PHYSICS

Answer any four problems. Do not turn in (partial) solutions for more than four problems. Each problem has the same weight.

1. Evaluate by contour integration the following integrals

(a)

$$I = \int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$$

(b)

$$I = \int_{-\infty}^{+\infty} \frac{x \sin \pi x}{x^2 + 2x + 5} dx$$

2. Assume that a system is represented by the following matrix in arrow form:

$$\begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} & \dots \\ H_{21} & H_{22} & 0 & 0 & \dots \\ H_{31} & 0 & H_{33} & 0 & \dots \\ H_{41} & 0 & 0 & H_{44} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

(a) Show that the exact eigenvalue λ for the first eigenvector $|1\rangle$ can be written as

$$\lambda = H_{11} + \sum_{n=2}^{\infty} \frac{H_{1n}H_{n1}}{\lambda - H_{nn}}$$

(b) Is there any physical application for which this formula might be relevant?

(c) How can we solve for λ ?

(d) How can we deal approximately with the infinite sum?

3. A matrix M is called **idempotent**, if $M^2 = M$.

(a) Give an example of an idempotent matrix that comes to mind.

(b) The following matrix is supposed to be idempotent. Unfortunately, one element has been lost during communication.

What is the lost element?

$$A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & ? & -3 \end{pmatrix}$$

(c) Show that, if $AB = A$ and $BA = B$, then A and B are idempotent.

4. Expand the function $f(z) = \frac{1}{z^2 - 1}$ in a Laurent series about the point $z = 1$ valid everywhere.

5. Two chemicals, A and B, react to form another chemical C. It is found that the rate at which C is formed varies as the product of the instantaneous amounts of chemicals A and B present. The formation requires 2 lb of A for each pound of B. If 10 lb of A and 20 lb of B are present initially, and if 6 lb of C are formed in 20 min, find the amount of chemical C at any time.