

# Physics Qualifying Examination

Friday, August 20, 2004  
August 20, 9 am - 12:30 pm

This examination has general questions from six areas of physics:

1. Mechanics
2. Thermodynamics
3. Electricity and Magnetism
4. Optics
5. Mathematical Physics
6. Modern Physics

Attempt all problems using a separate sheet of paper for each.

Grading of the examination will be based on your approach to solving the problem, and on your application of basic physical principles. All problems are weighed equally.

## FUNDAMENTAL PHYSICAL CONSTANTS

Planck constant  $h = 6.626076 \times 10^{-34} \text{ J}\cdot\text{s}$

Boltzmann constant  $k_B = 1.38066 \times 10^{-23} \text{ J/K}$

Elementary charge  $e = 1.60217733 \times 10^{-19} \text{ C}$

Avogadro number  $N_A = 6.0221367 \times 10^{23} \text{ particles/mol}$

Speed of light  $c = 2.99792458 \times 10^8 \text{ m/s}$

Permeability of vacuum  $\mu_0 = 4\pi \times 10^{-7} \text{ T}^2\cdot\text{m}^3/\text{J}$

Permittivity of vacuum  $\epsilon_0 = 8.854187817 \times 10^{-12} \text{ C}^2/\text{J}\cdot\text{m}$

Fine structure constant  $\alpha = 1/137.0359895$

Electron rest mass  $m_e = 9.1093897 \times 10^{-31} \text{ kg}$

Proton rest mass  $m_p = 1.6726231 \times 10^{-27} \text{ kg}$

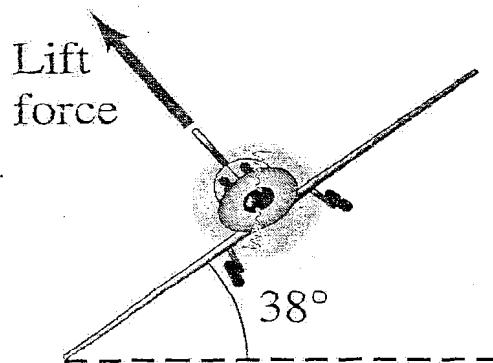
Gas constant  $R = 8.31451 \text{ m}^2\text{kg/s}^2\cdot\text{K}\cdot\text{mol}$

Gravitational constant  $G = 6.673 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$

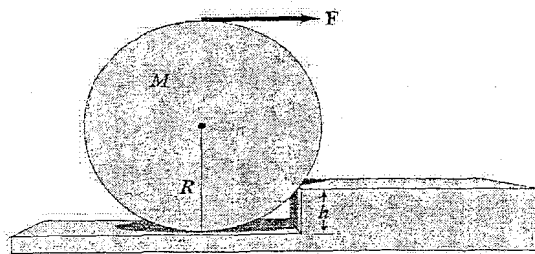
Acceleration due to gravity  $g = 9.80665 \text{ m/s}^2$

## I. Mechanics

A) An airplane traveling at 520 km/h needs to reverse its course. The pilot decides to accomplish this by banking the wings at an angle of  $38^\circ$ . Find the time needed to reverse course. *Hint: Assume an aerodynamic "lift" force that acts perpendicularly to the flat wings; see Fig. below.*



B) What minimum horizontal force  $F$  is needed to pull a wheel of radius  $R$  and mass  $M$  over a step of height  $h$  as shown in Fig. below ( $R > h$ ). Assume the force is applied at the top edge as shown.



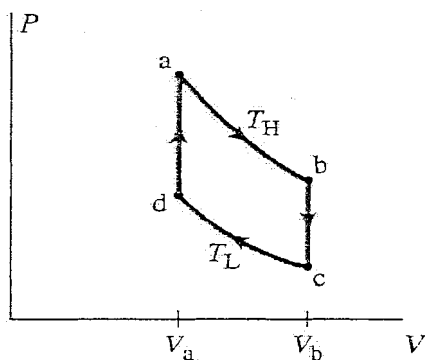
## II. Thermodynamics

A) The temperature of an ideal gas is increased from  $120^{\circ}\text{C}$  to  $290^{\circ}\text{C}$  while the volume and the number of moles stay constant.

(a) By what factor does the pressure change?

(b) By what factor does the RMS velocity of atoms  $v_{rms}$  change?

B) The Stirling cycle, shown in the Fig. below, is useful to describe external combustion engines as well as solar-power systems. Find the efficiency of the cycle in terms of the parameters shown, assuming a monoatomic gas as the working substance. The processes  $ab$  and  $cd$  are isothermal whereas  $bc$  and  $da$  are at constant volume. How does it compare to the Carnot efficiency?

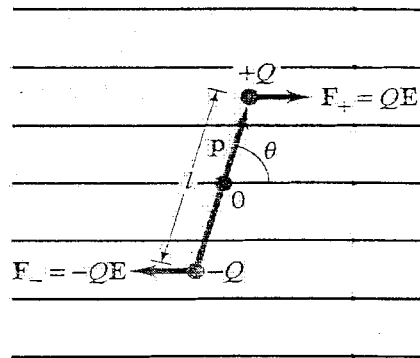


### III. Electricity and Magnetism

A) An electric dipole, of dipole moment  $p$  and moment of inertia  $I$ , is placed in a uniform electric field  $\mathbf{E}$ .

(a) If displaced by an angle  $\theta$  as shown in the Fig. and released, under what conditions will it oscillate simple harmonic motion?

(b) What will be its frequency?



B) A straight stream of protons passes a given point in space at a rate of  $1.5 \times 10^9$  protons/second. What magnetic field do they produce 2.0 m from the beam.

#### IV. Optics

A) A 1.00-cm-high object is placed 10.0 cm from a concave mirror whose radius of curvature is 30.0 cm.

- (a) Draw a ray diagram to locate (approximately) the position of the image
- (b) Determine the position of the image and the magnification analytically.

B) Light of wavelength  $\lambda$  strikes a screen containing two slits a distance  $d$  apart at an angle  $\theta_i$  to the normal. Determine the angle  $\theta_m$  at which the  $m^{\text{th}}$ -order maximum occurs.

## V. Mathematical Physics

A) Show that if a real function  $f(x)$  is expanded in a complex exponential Fourier series  $\sum_{-\infty}^{\infty} c_n e^{inx}$ , then  $c_{-n} = (c_n)^*$ .

B) Solve differential equation

$$xy'' + y' = 4x.$$

C) Find the eigenvalues and eigenfunctions of the matrix

$$\begin{pmatrix} -1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 3 \end{pmatrix}.$$

## VI. Modern Physics

A) An atomic clock is taken to the North Pole, while another stays at the Equator. How far will they be out of synchronization after a year has elapsed? (radius of the Earth is  $6.38 \times 10^6$  m.)

B) An electron trapped in an infinitely deep square well has a ground-state energy  $E = 8.0$  eV.

- (a) What is the longest wave-length photon this system can emit?
- (b) What is the width of the box?

END of problems